BENDABLE MIRRORS FOR SYNCHROTRON RADIATION BEAM LINES

BY MALCOLM R. HOWELLS

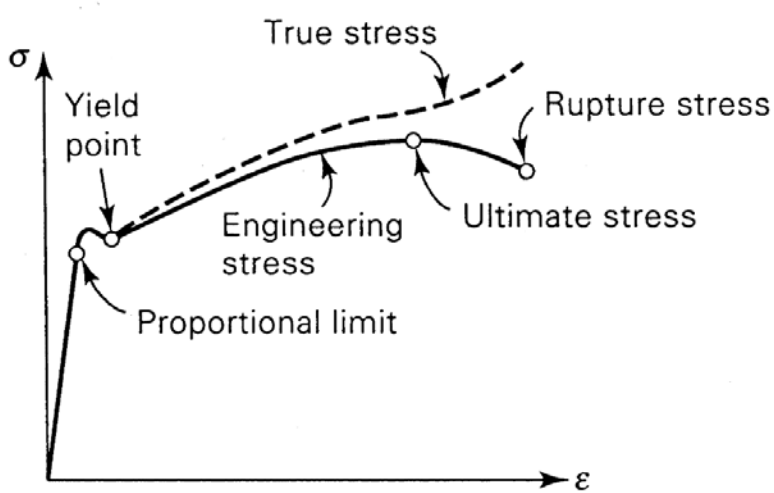
ADVANCED LIGHT SOURCE

BENDING AS A WAY OF MAKING MIRRORS

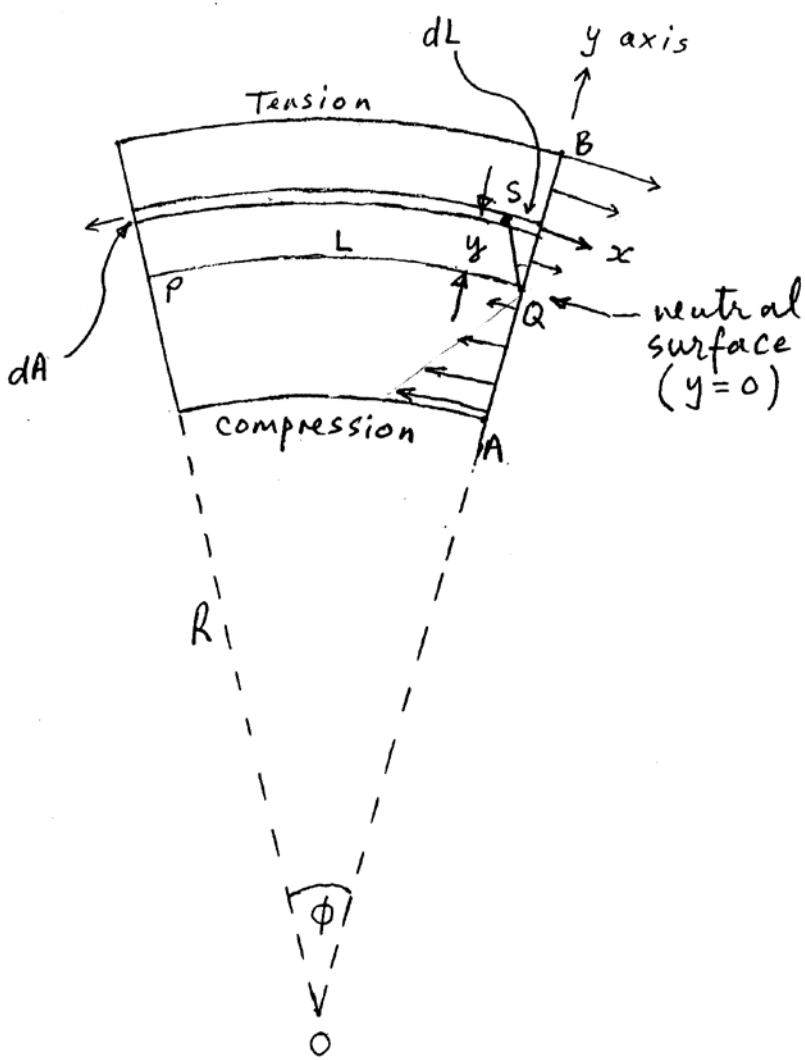
- Allows mirrors to have adjustable focal length etc as well as variable incidence angles and energy filtering properties etc
- Substrates are lighter and cheaper and not necessarily harder to polish
- Mirrors are polished as *flats* - many benefits such as repeated use of the same laps, better figure and finish for a given effort, standard measurement tools work well
- Because of their adjustability bent mirrors often have *better* figure than their rigid counterparts especially when the amount of bend is small
- By profiling the edges of an otherwise uniform beam one can get shapes such as elliptical cylinders that would be prohibitively difficult to make any other way
- The down side is that a bending machine of some sort has to be built - however for mirrors to be bent to *fixed* radius (the majority) the designs can be quite simple

BASICS OF BEAM BENDING

- Galileo's problem (1638) - not solvable without a stress-strain relationship
- Sir Robert Hooke (1678)
  - Define stress (σ) = force per unit area
  - Define strain (ε) = fractional change in length
  - Hooke's law: $\varepsilon_x = \sigma_x / E$ where E is the elastic modulus
  - Hookes law limited to strains small enough for a linear stress/strain relationship to apply



INTRODUCTION TO BEAM THEORY



Draw QS parallel to OP, let OP = R

$$L = R\phi \qquad \Delta L = y\phi$$

$$\varepsilon = \frac{\Delta L}{L} = \frac{y\phi}{L} = \frac{y\phi}{R\phi} = \frac{y}{R} \;\rightarrow\; \sigma = \frac{Ey}{R}$$

Boundary conditions:

1. Equilibrium: $\displaystyle\int_{AB} \sigma_x dA = \frac{E}{R}\int_{AB} y dA = 0$

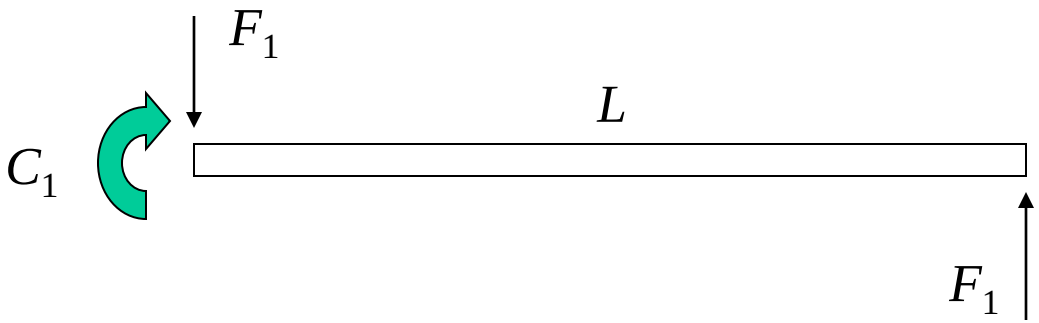
$\therefore$ neutral surface is centroidal

2. Moment about the neutral surface equals the bending moment M

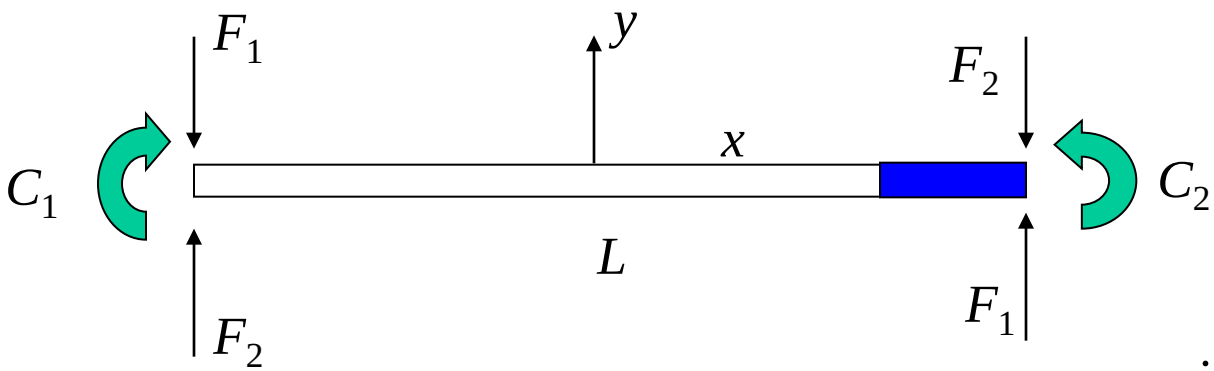
$$M = \int_{AB} y\sigma_x dA = \frac{E}{R}\int_{AB} y^2 dA = \frac{EI}{R} \qquad \boxed{EI\frac{d^2y}{dx^2} = M}$$

where I = moment of inertia of the section and the DE is the Bernoulli – Euler equation

BENDING MOMENT FOR UNEQUAL COUPLES



$$C_1 = F_1 L$$



$$C_2 = F_2 L$$

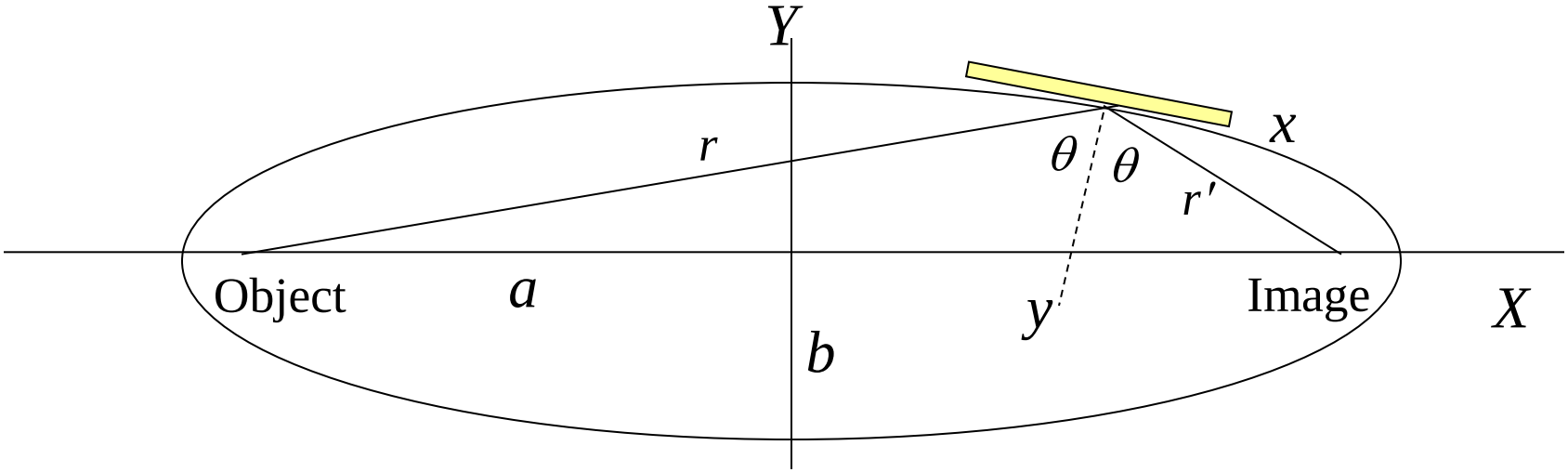
$$\therefore \quad F_1 - F_2 = \frac{C_1 - C_2}{L}$$

The condition for equilibrium of the blue segment is thus

$$M(x) = C_2 + (F_1 - F_2)\left[\frac{L}{2} - x\right]$$

$$= \frac{C_1 + C_2}{2} - \frac{C_1 - C_2}{L} x \quad (\text{with } C_1 \text{ at } -x \text{ and } C_1 > C_2\ )$$

ELLIPTICAL MIRRORS



$$\frac{X^2}{a^2}+\frac{Y^2}{b^2}=1$$

$$y=a_2x^2+a_3x^3+a_4x^4+\mathsf{K}$$

$$\frac{d^2y}{dx^2}=2a_2+6a_3x+12a_4x^2+\mathsf{K}$$

$$EI\frac{d^2y}{dx^2}=\frac{C_1+C_2}{2}-\frac{C_1-C_2}{L}x$$

Equating coefficients of x^0 and x^1

we get a cubic approximation

$$C_1+C_2=4EI_0a_2 \qquad C_1-C_2=-6EI_0La_3$$

Since generically $I=wt^3/12$ we can

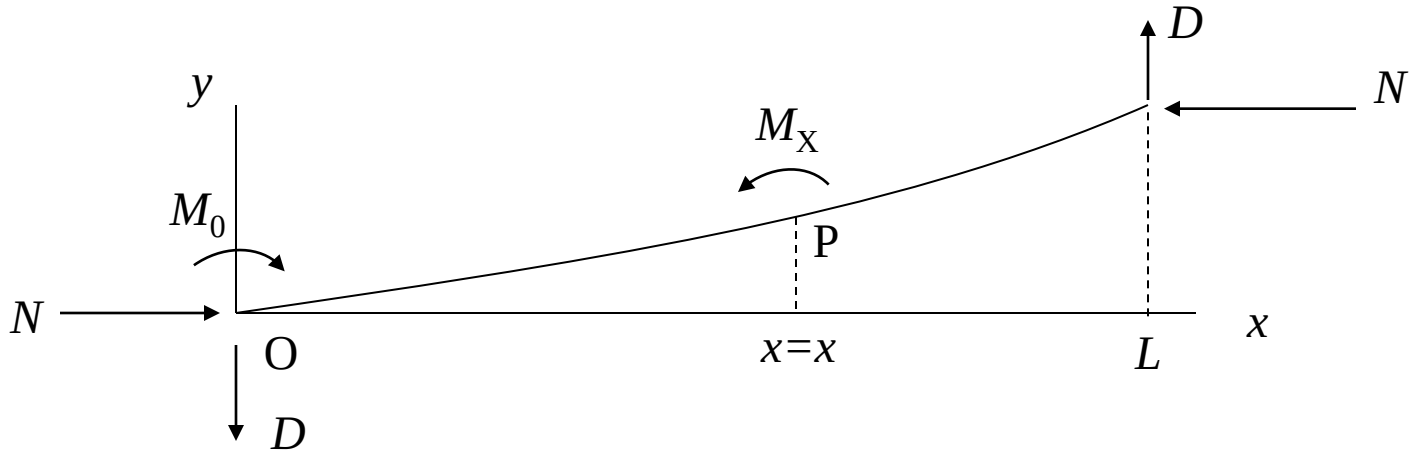
get a nominally exact ellipse if

$$w(x)=\frac{w_0\left(2a_2+6a_3x\right)}{2a_2+6a_3x+12a_4x^2+\ldots}$$

Last equation comes from using the same couples and setting

$$I_0\left[\frac{d^2y}{dx^2}\right]_{\text{cubic}}=I(x)\left[\frac{d^2y}{dx^2}\right]_{\text{exact}}$$

MORE GENERAL BEAM THEORY



Equilibrium of OP

$M(x) = M_0 - Dx - Ny$ then using "$EI/R = M$"

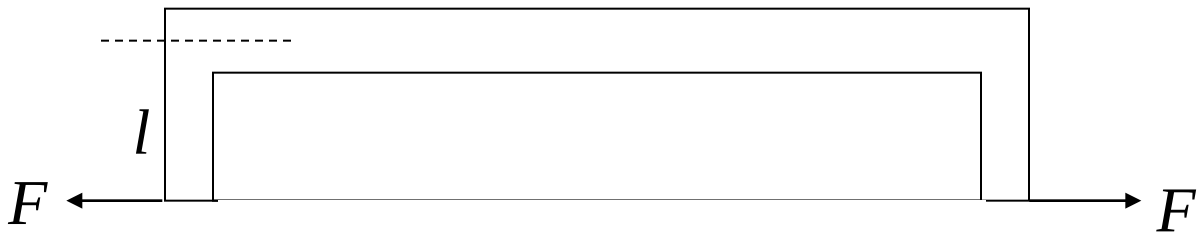
$$\frac{d^2y}{dx^2} + \frac{N}{EI}y = \frac{1}{EI}(M_0 - Dx)$$ now let $N/EI = q^2$ and the solutions are

N compressive: $$y = -\frac{D}{qN}(qx - \sin qx) + \frac{M_0}{N}(1 - \cos qx)$$

N tensile: $$y = \frac{D}{qN}(qx - \sinh qx) + \frac{M_0}{N}(1 - \cosh qx)$$

MIRROR BENDERS THAT ARE IN TENSION

Example: equal couples (circular-cylinder mirror)



$$M_0 = Fl, \quad D = 0, \quad N = F$$

$$y = \frac{M_0}{N}(1 - \cosh qx)$$

By definition of q and $M_0 / EI = 1/R_0$

$$y = \frac{x^2}{2R_0}\left\{\frac{\sinh\left(\frac{qx}{2}\right)}{\frac{qx}{2}}\right\}^2$$

Notes:

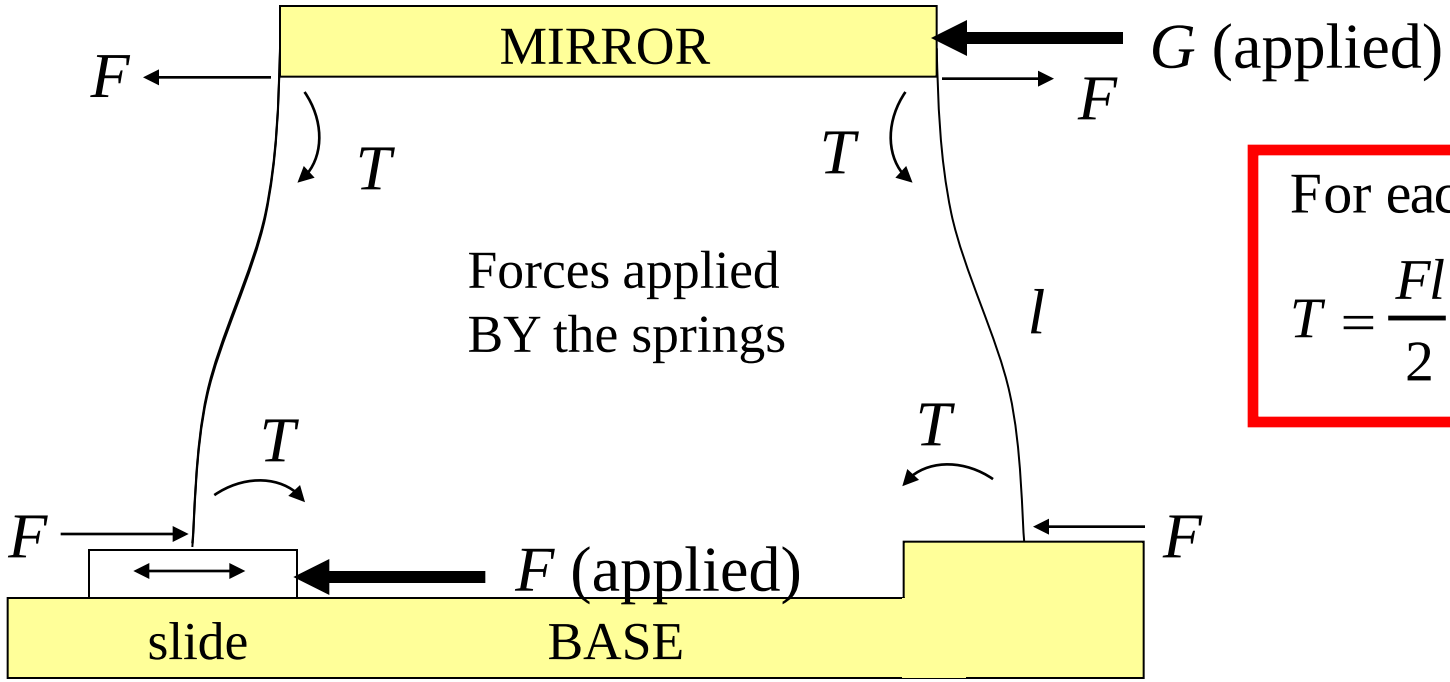
When $N = 0 \quad \{\ \} = 1 \rightarrow$ circle

$$\frac{dy}{dx} = \frac{x}{R_0}\left[\frac{\sinh qx}{qx}\right]$$

Max slope error is at the ends

and equals $1 - \dfrac{\sinh(qL/2)}{qL/2}$

ONE LEAF-SPRING BENDER SCHEME

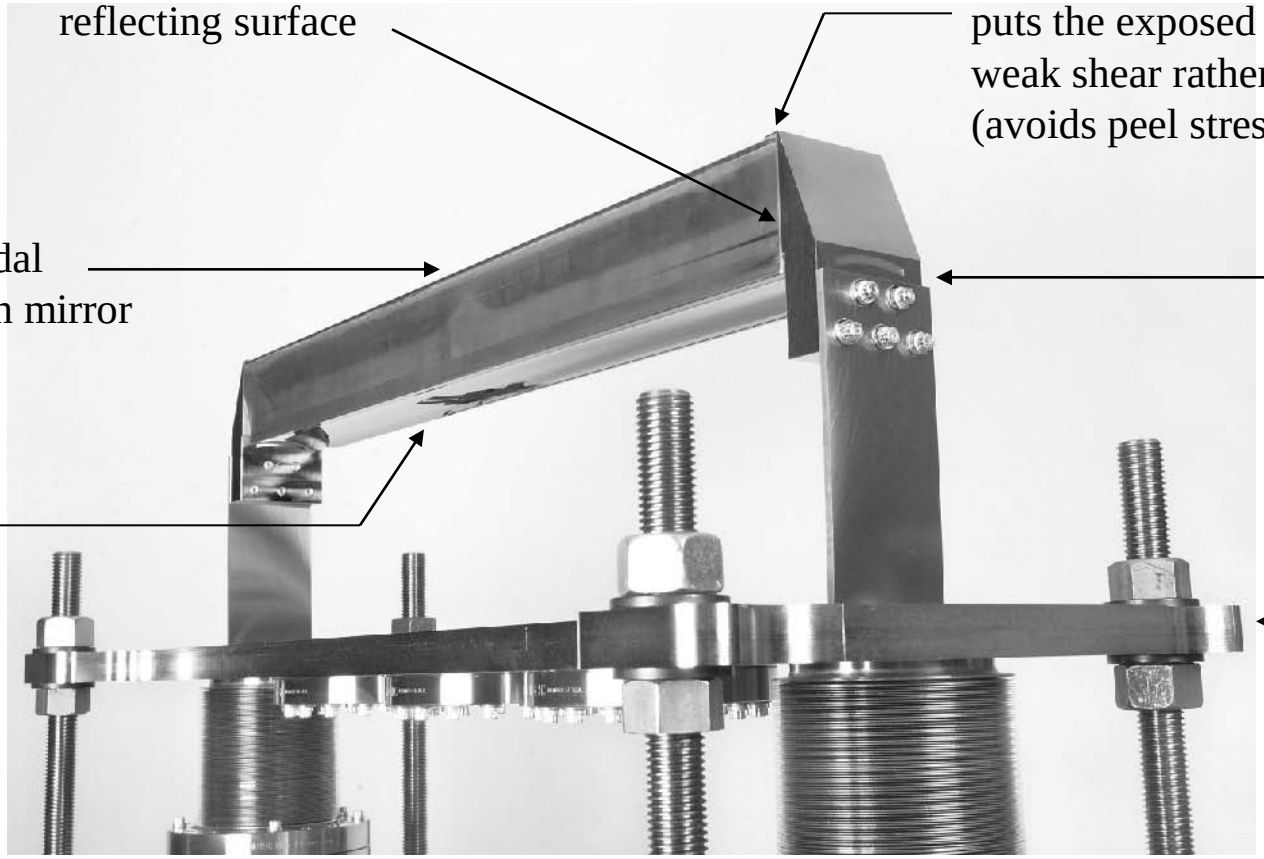


For each spring:

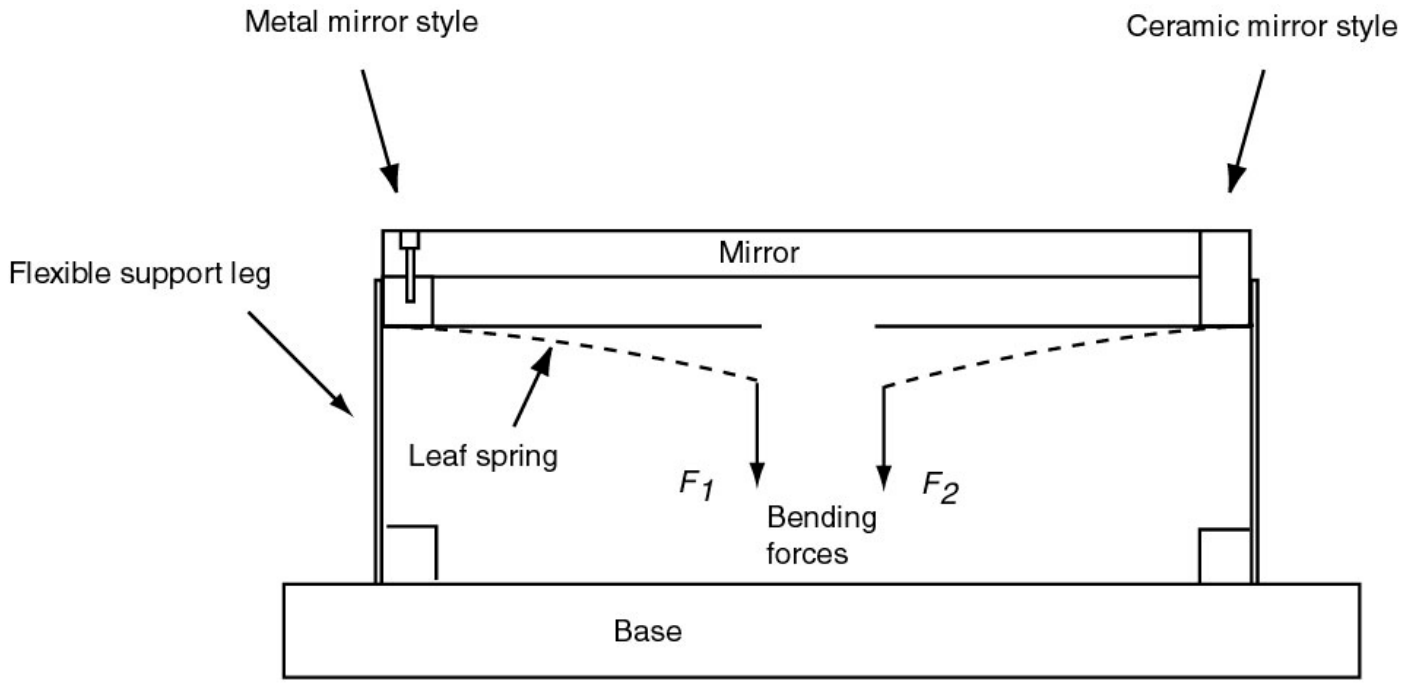
$$T = \frac{Fl}{2}$$

<u>Apply *F* first:</u>
We get equal couples ±*T*, circular mirror, radius determined by *F*
<u>Then apply *G*:</u>
Now the couples are *T*–*C* on left, –(*T*+*C*) on right where *C*=*Gl* /4
Therefore *F* controls focal length, *G* controls abberation correction independently
Mirror is in tension by *F*

PRACTICAL EXAMPLE OF "S" BEND DESIGN



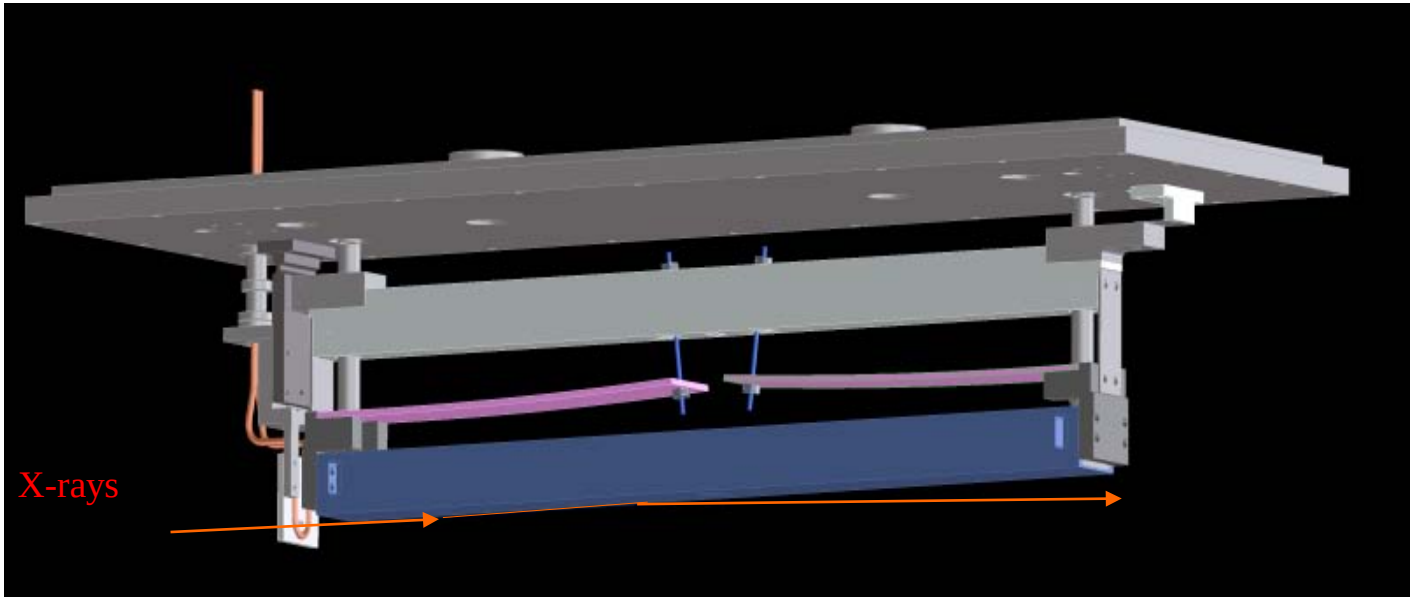
ALS LATEST MIRROR BENDER DESIGN



Pro's and con's:

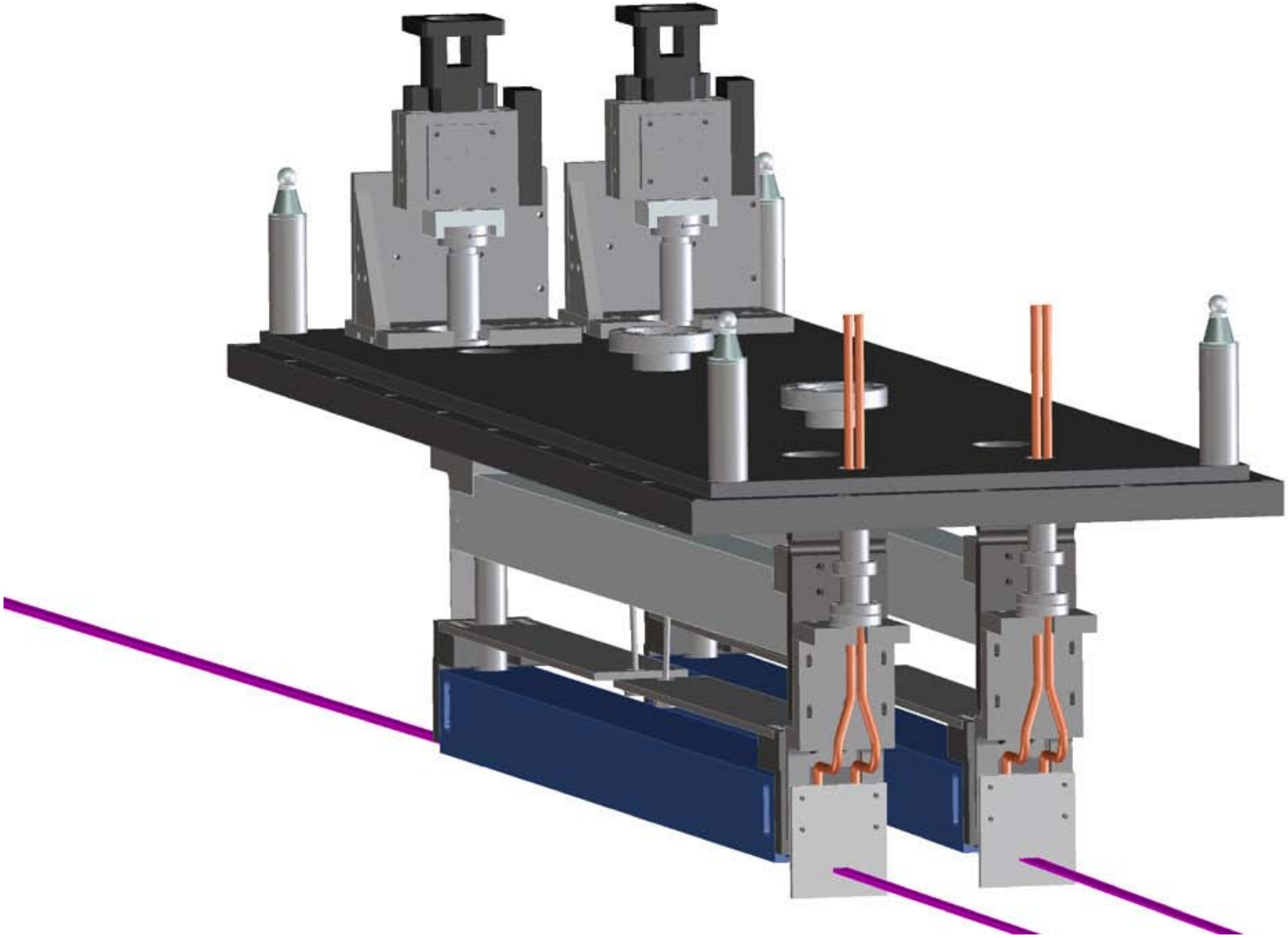
- All flexural, i. e. no moving parts such as slides in vacuum
- Does not put the mirror in tension
- Compatible with both metal and ceramic substrates and with cooling
- Focus and aberration correction no longer independent
- Flexible support legs need not be all that flexible

INVAR PX MIRROR DESIGN CONCEPT

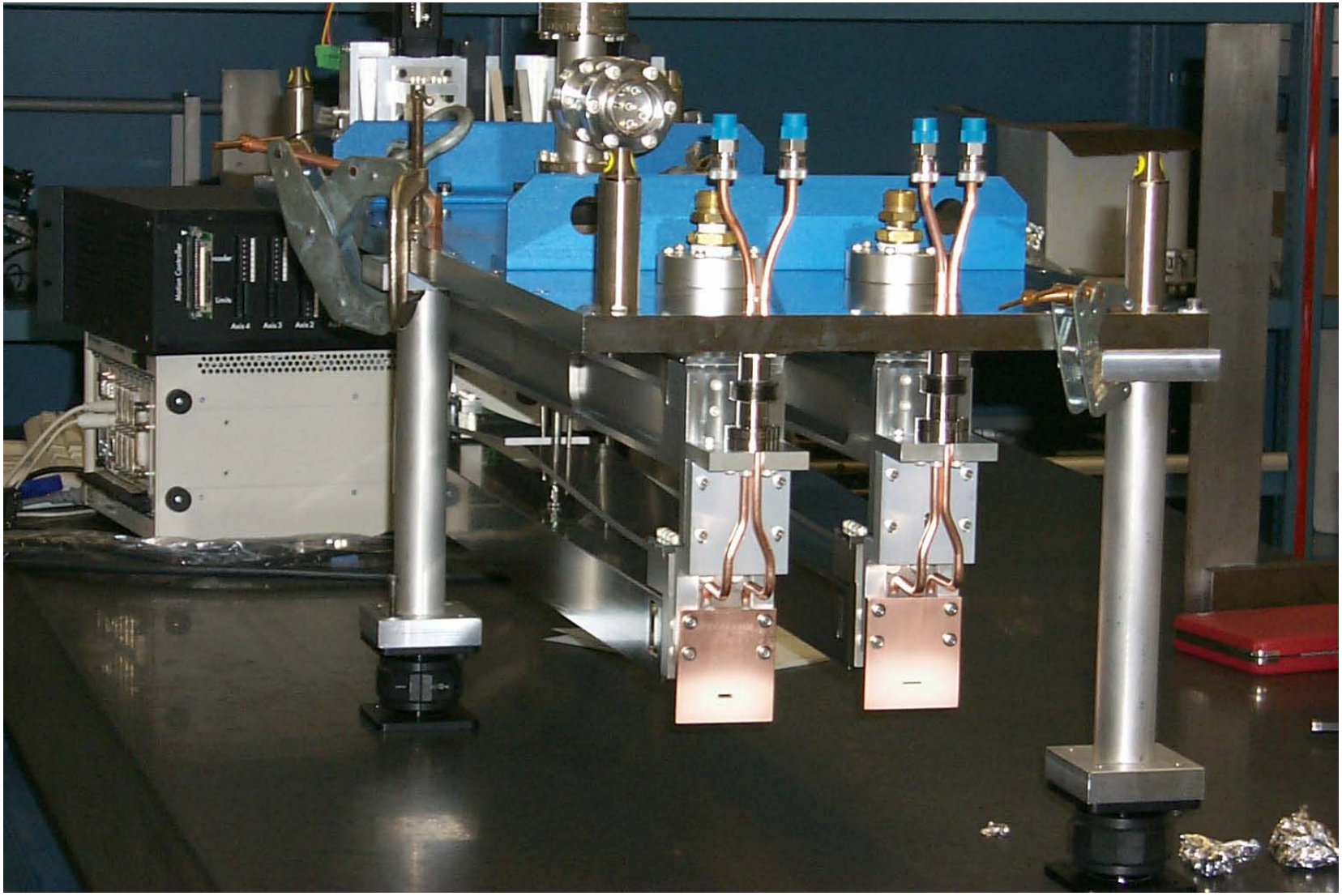


- Reflecting surface faces downward
- Bending forces are applied by adjustable flexible leaf springs
- Mirror shape is a cubic approximation to the ideal parabolic cylinder that collimates the beam for entry into the double crystal monochromator

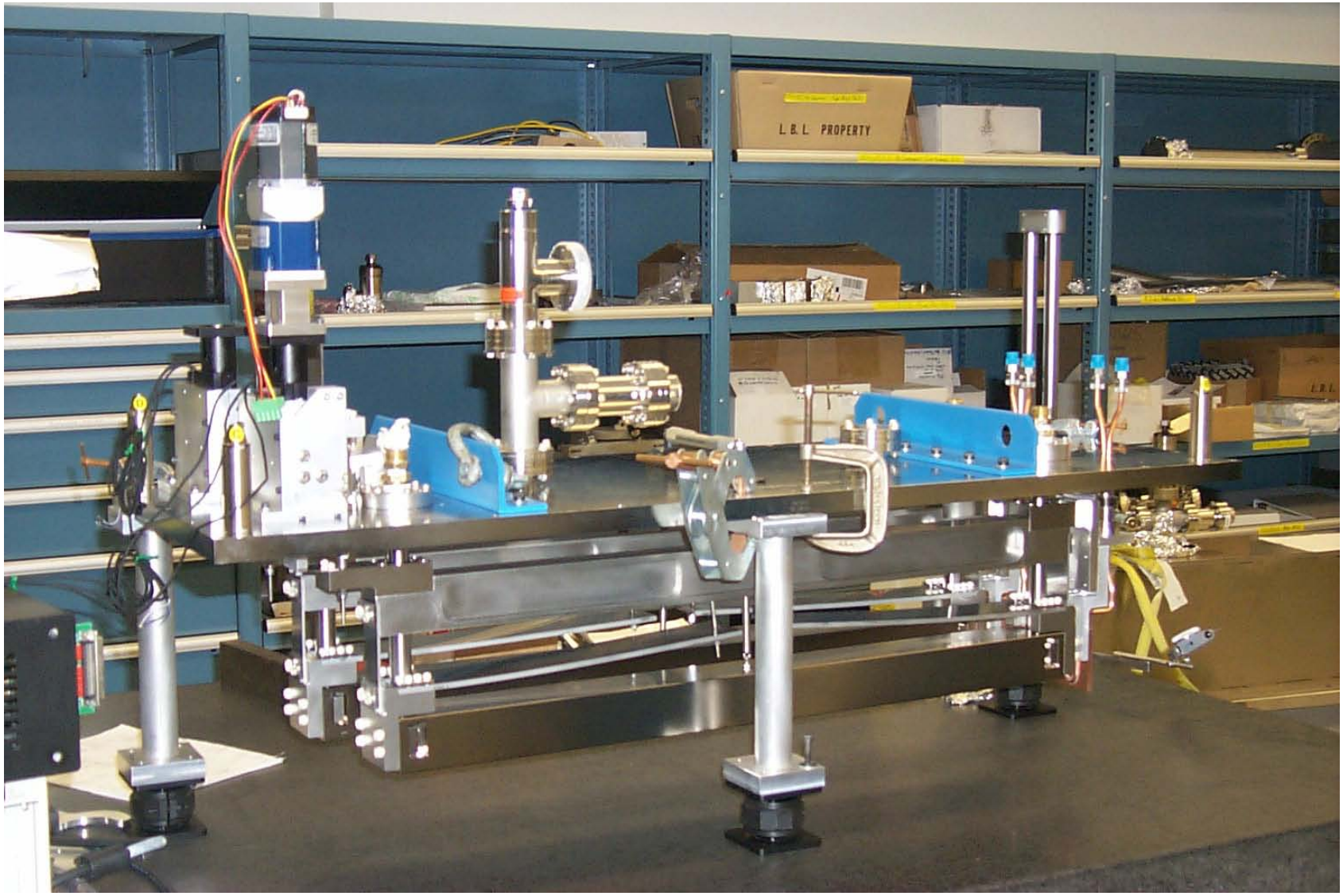
MIRROR PAIR TO BE 6.5 m FROM SUPERBEND



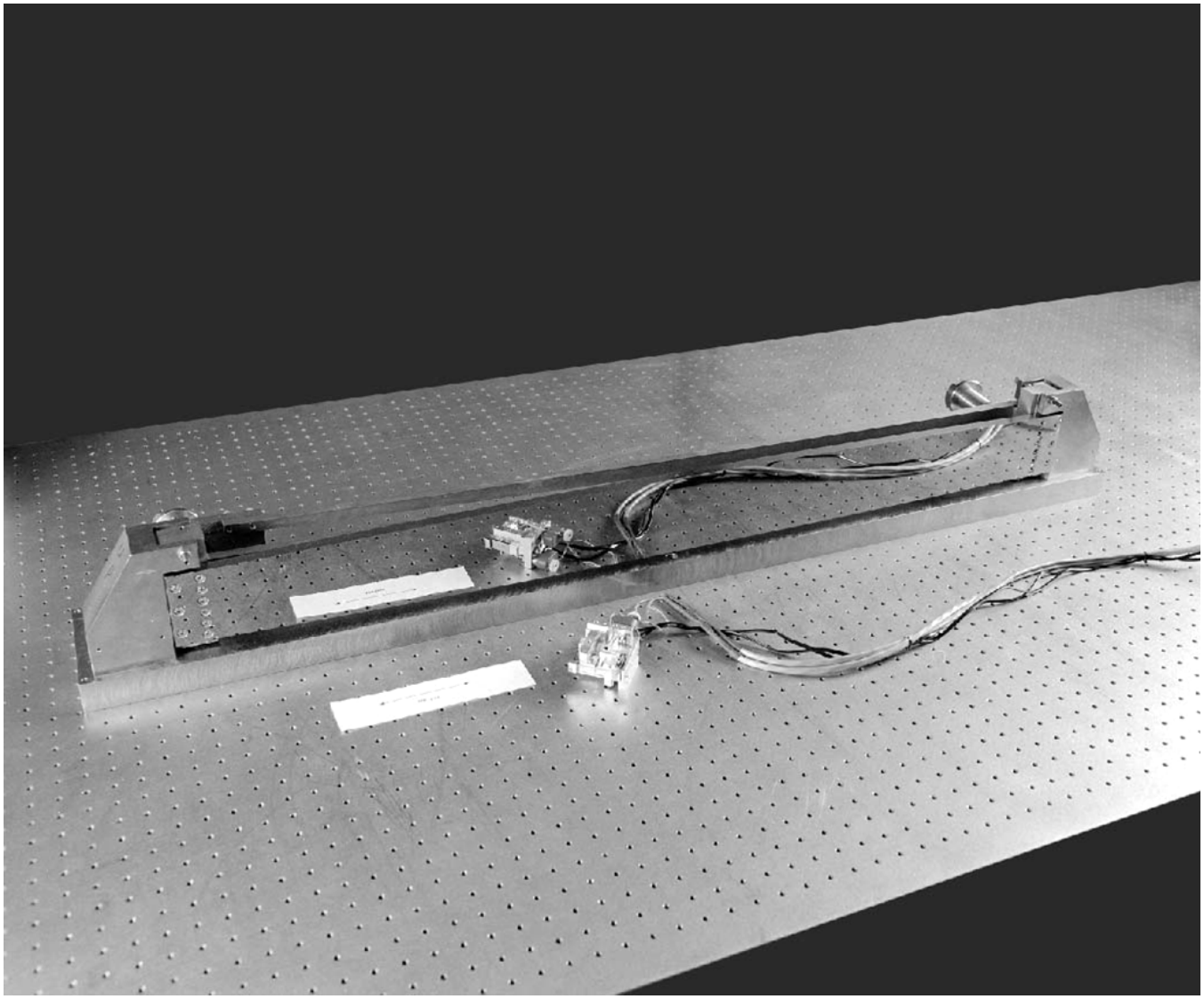
MIRROR PAIR DURING ALIGNMENT



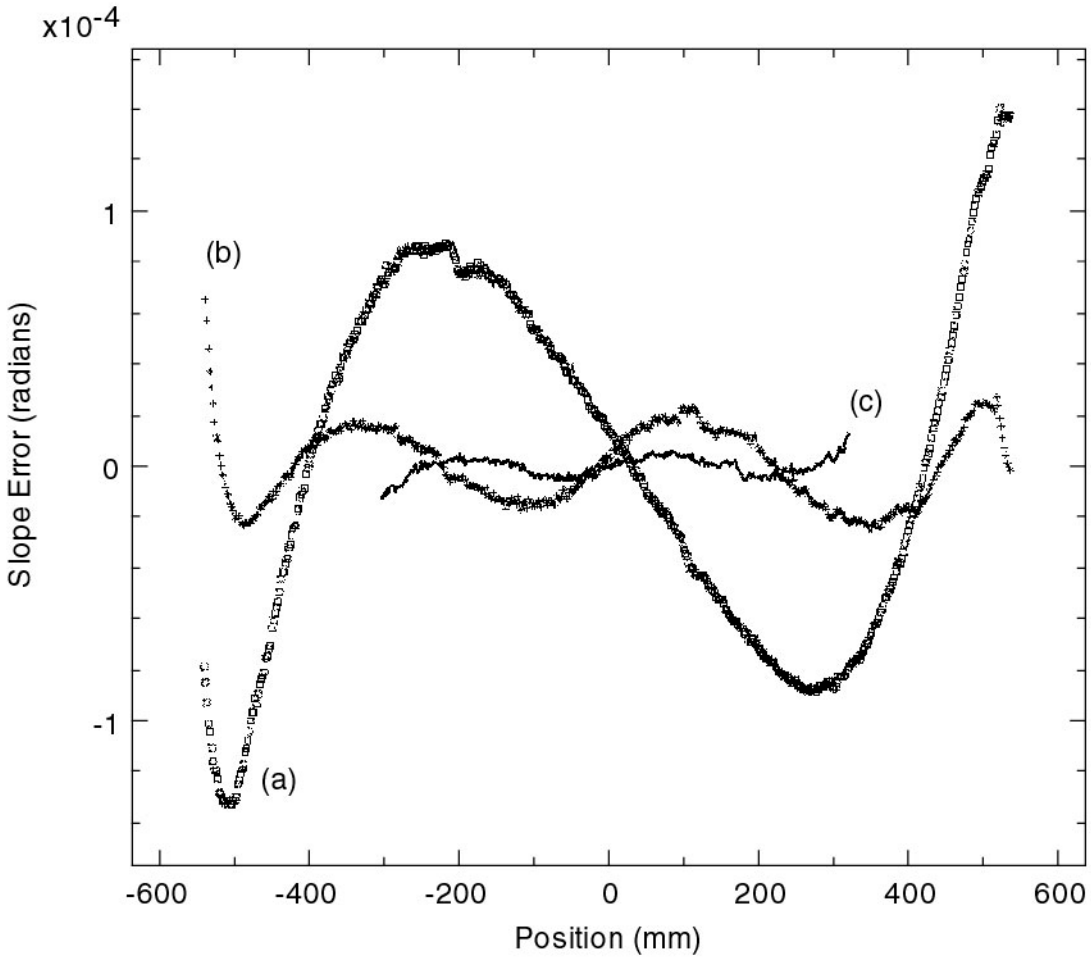
TWO INVAR MIRRORS READY TO INSTALL



THE LARGEST AND SMALLEST ALS MIRRORS



PEEM CONDENSER LTP DATA



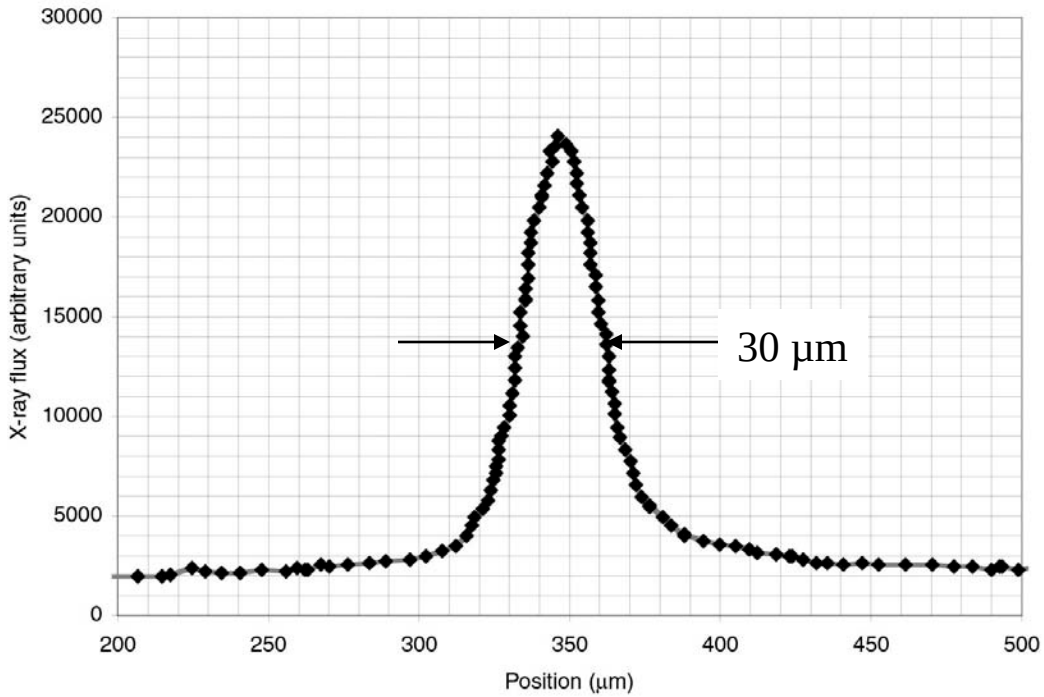
(a) Optimization by tuning intentional defocus

(b) Combination of (a) with help from a point load at the center

(c) Similar to (b) but applied over the center 60% of the mirror

(d) After polishing the mirror had 100 µr slope error at each end of "spherical aberration" - (see Howells et al 2000 Opt Eng paper ¶ 9.2.2 and 11.1) - this came from an unintended curvature of the delivered optic - nevertheless the above strategies reduced this to

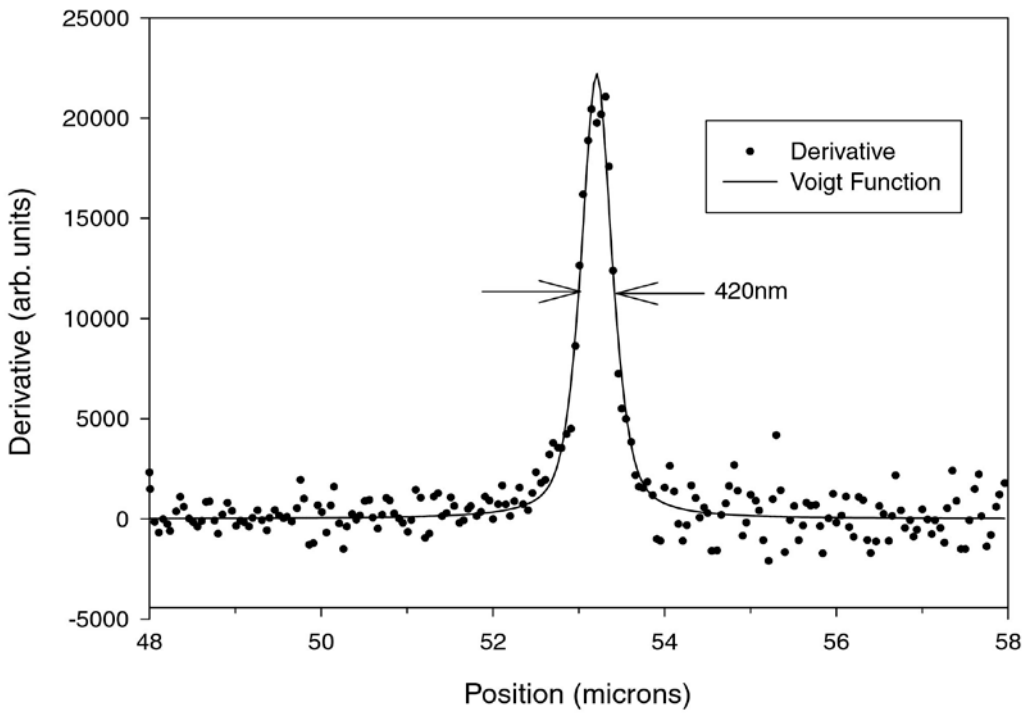
X-RAY SPOT FROM THE PEEM CONDENSER



- Elliptical cylinder mirror with 10:1 demagnification
- Produces a good image of the ALS horizontal source size (FWHM=300µm)
- Mirror length = 1.25 m
- Slope errors:
  3 µr at 60% aperture
  14 µr at 100% aperture
- Material: mild steel type 1006
- Assembly: nut and bolt to underside
- Special requirement: tension not allowed

MICRO X-RAY DIFFRACTION MIRROR

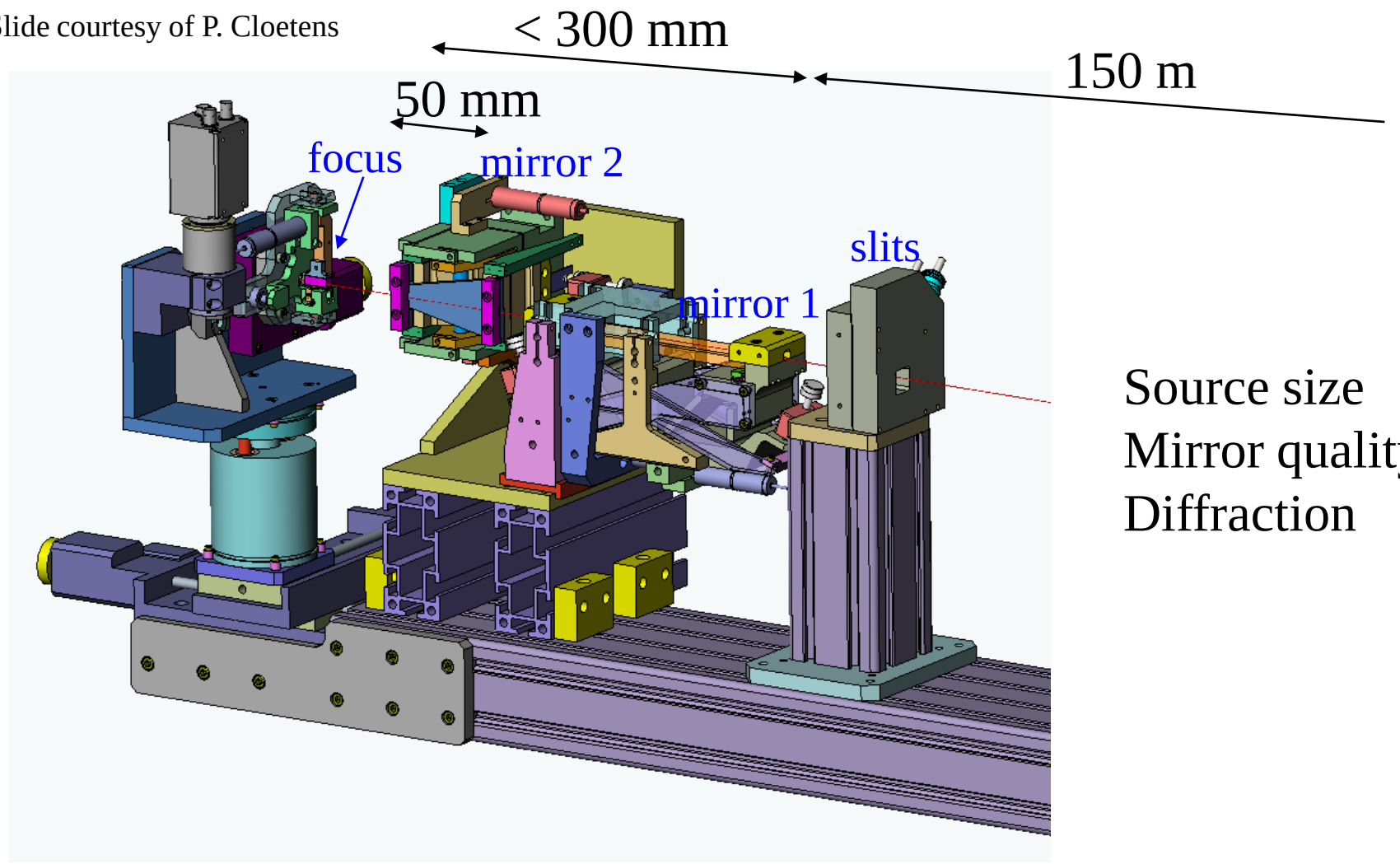
Macdowell 1998



- Elliptical cylinder with demagnification 310:1
- Length: 4 cm
- Material: ULE
- Attachments: glue
- Slope error: 0.6 µr
- Grazing angle: 6 mr
- Smallest spot size yet achieved by ALS group

Kirkpatrick-Baez focusing

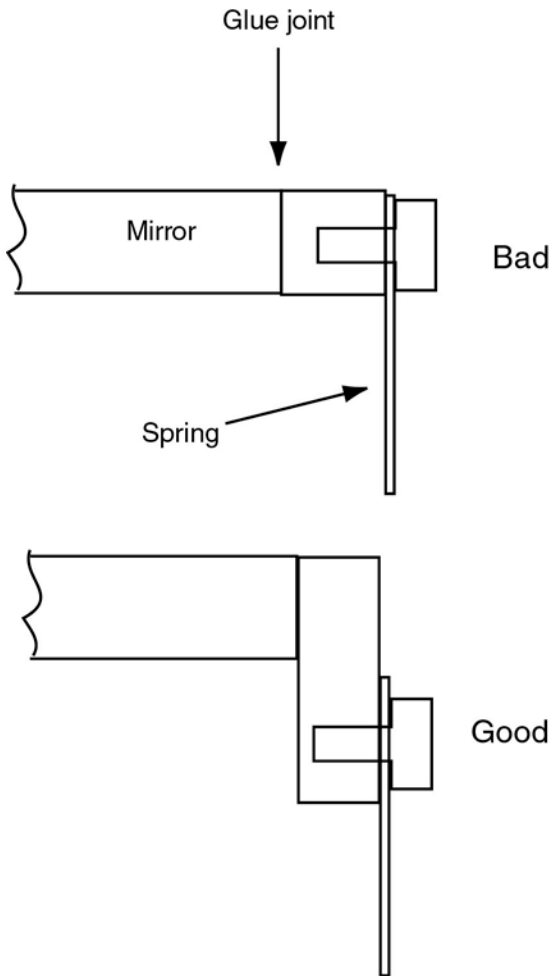
Slide courtesy of P. Cloetens



Source size
Mirror quality
Diffraction

90 nm x 90 nm focus (up to 10^{12} ph/s @ $\Delta E/E=10^{-2}$)

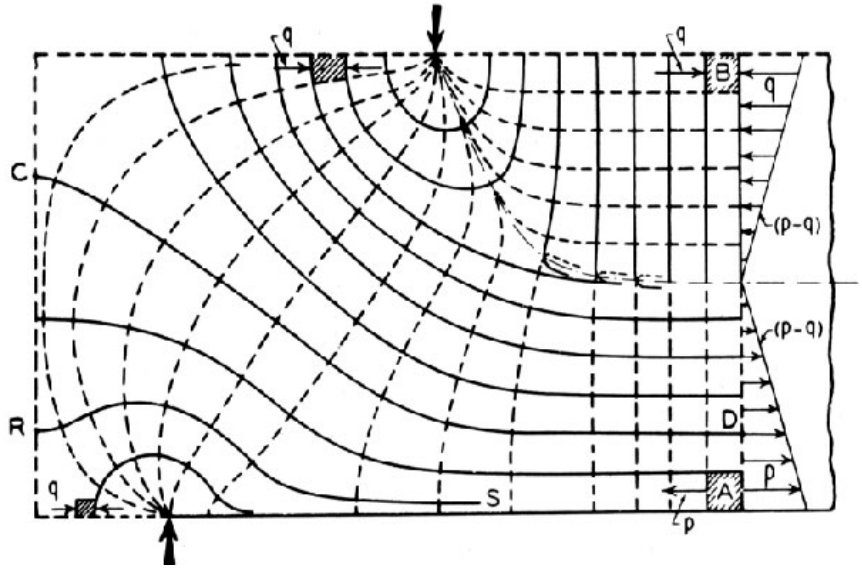
GENERAL TECHNIQUE FOR SUBSTRATES AND ATTACHEMENTS

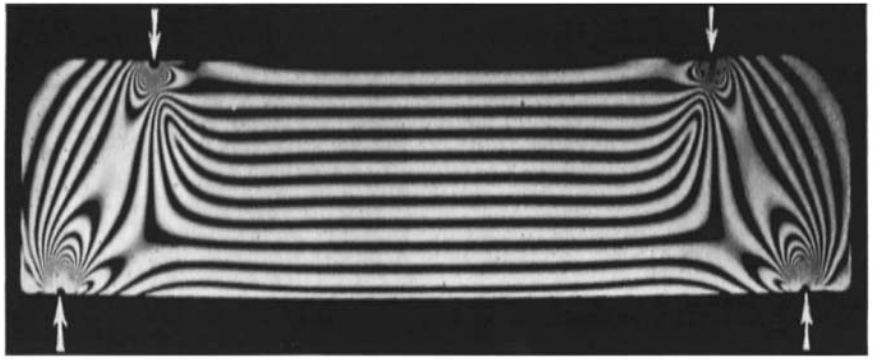


- Substrates should be ground (gently) and lapped back and front and be flat and parallel to about a part in 1000
- Glueing, soldering or bolting to the front or back of the mirror often cases distortions - can work well if both mating surfaces are lapped flat
- Attachement to the end surface works better
- Nevertheless the bolts will produce distortions which need to be isolated even from the end of the mirror (see left)
- Glue joints (we use Dexter Hysol 9309.3NA) are popular for ceramic mirrors but they are hard to do for mirrors below about 2-3 mm thick
- Assembly may produce a twist in the mirror - there needs to be a sensitive (perhaps flexural) twist corrector in the system - twist errors can be removed using tilt sensors

THE DETAILS OF APPLYING THE END COUPLES

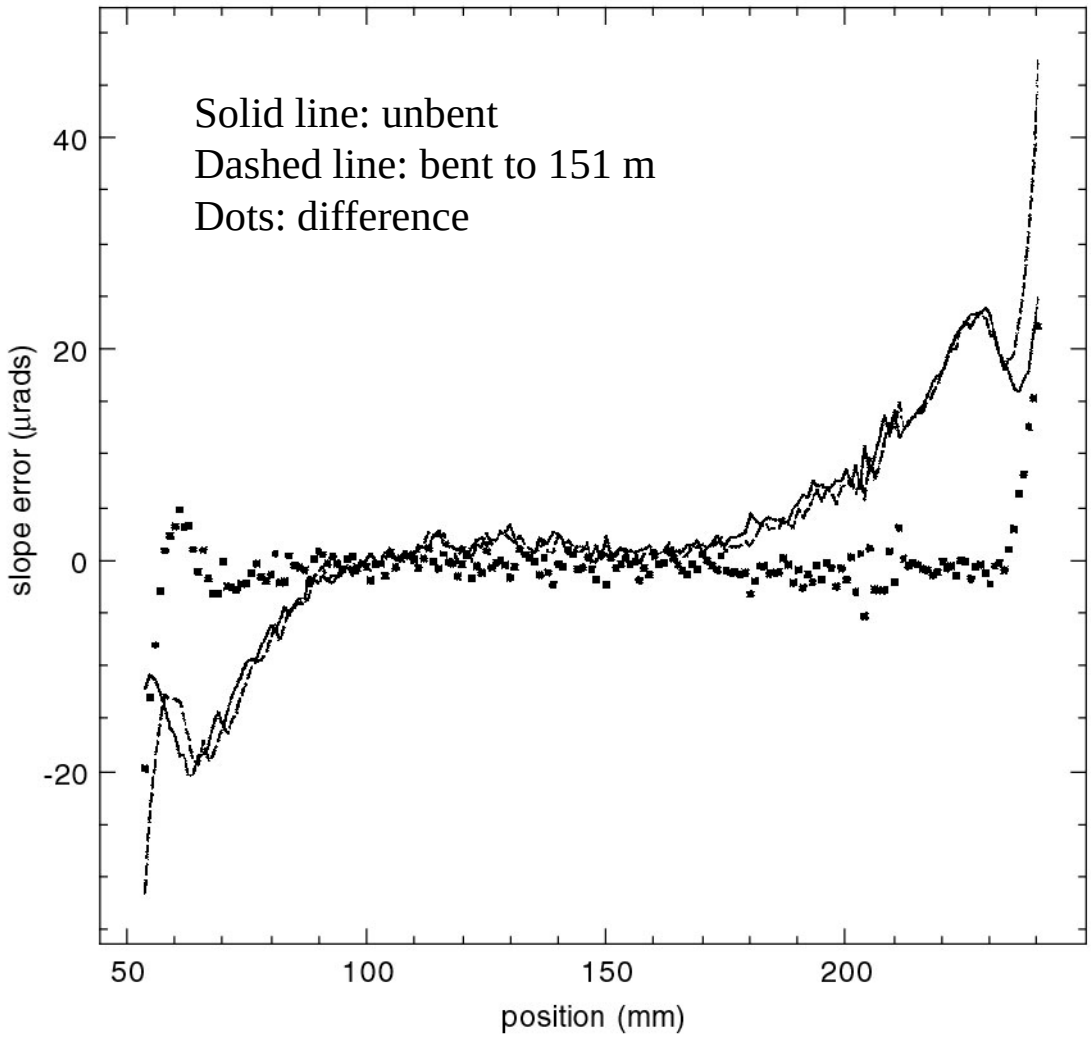
Frocht 1941



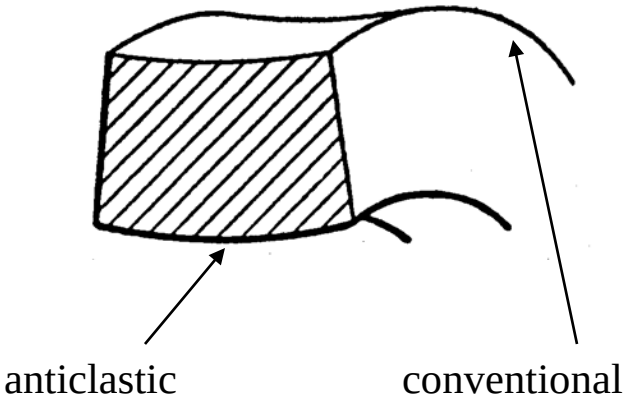


- The goal is a smooth circular bend but this is almost never achieved
- Near the ends complicated stress patterns are often produced
- However, at points more than about one thickness away from the applied forces, the local end stresses are unimportant and only the integrated couple matters (example of St. Venant's principle)
- These examples, one calculated and one measured by photoelasticity, show a reversion to parallel equispaced stress lines (pure bending) at about one thickness
- This tells us how far the end effects of bent mirrors should extend

COMPLEX ERRORS DUE TO ANTICLASTIC EFFECTS



- Generally a conventional curvature C leads to anticlastic curvature Cν where ν is Poisson's ratio (= about 1/3)
- Left is data from a mirror with bending fixtures glued to underside
- Why does the error extend so far from the end?
- Glue shrinkage driving *inverse* anticlastic effect with consequent longitudinal curvature extending about one *width* rather than one thickness



GENERAL ADVANTAGES OF LEAF-SPRING BENDERS

- The focal length and aberration correction may be controllable independently
- The leaf springs are weak springs compared to the mirror. Therefore they act as “spring demagnifiers” and allow micron-scale changes to the mirror to be driven by centimeter-scale movements of the springs
- The system is thus tolerant of errors due to manufacturing or thermal expansion so long as the errors are small compared the driver motions (compare with PZT-driven systems - displacement transducers versus force transducers)
- Leaf strings have low stress compared to "notch" hinges for a given amount of motion. However they still have much higher stress than the mirror and strong low-creep materials such as precipitation-hardened S/S (17-4 PH etc) may be needed.
- These bender schemes have a base (which can be a vacuum flange) and forces are applied relative to the base.

SUMMARY OF ALS ELLIPTICAL BENDERS

	μ-XPS (horiz)	μ-XPS (vert)	PEEM CONDENSER	7.3.3 CONDENSER	μ-XRD (vert)	μ-XRD (horiz)
Initial shape	Flat	Flat	Flat	Cylinder, radius 87.1mm	Flat	Flat
Final shape	Elliptical cyl	Elliptical cyl	Elliptical cyl	Toroid	Cubic cyl	Elliptical cyl
r (m)	4	3.88	20	16	30.6	31
r' (m)	0.1	0.22	1.85	16	0.5	0.1
Grazing angle (°)	1.6	1.6	2.5	0.31	0.33	0.33
Clear aperture (mm^2)	60×25	110×38	1010×100	600	163×42	40×10
Thickness (mm)	3	6.7	15	32	9.52	4
Min-max bending radius (m)	4.5, 9.7	10.3, 20.0	49.8, 107.5	2963.0	130.1, 211.2	28.7, 40.4
Achieved rms slope error before bending (μr)	2	3	3 (60% aperture) 14 (100% aperture)	1.0	1.4	0.6
Achieved finish (Å rms)	3.0	4.0	7	7.2	<1	5
Measured x-ray spot size FWHM (μm)	1.0	1.2	30 (100% aperture)	50	0.8	0.4*
Material	17-4 PH stainless steel	17-4 PH stainless steel	Mild steel (1006)	Silicon	ULE	ULE
Polisher	Dallas Optical Systems	Dallas Optical Systems	Boeing North American	Frank Cook	General Optics	Boeing North American
Attachments	Nut and bolt	Nut and bolt	Nut and bolt	Glue	Glue	Glue
Special challenges	Extreme curvature		Avoid tension, large size	Figure accuracy		Figure accuracy