Design of a flexural hinge mechanism for Turbo Slits

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Flexural hinges

Monolithic

Clamped

Living hinge
Necksafe
Case study

- I20 turbo slit flexure
- Requirements
  - Vertical slits scanned across horizontal fan (80 mm)
  - Light weight / small size due to requirements of fast scanning
Approaches to the design of flexural hinge mechanisms

- Analytic equations based on a simplified model of the hinge
- FEA analysis of the hinge as an isolated component
- FEA analysis of the overall design
- Empirical values based on previous designs
- Mechanism analysis
**Simplified hinge model**

Angular Deflections $\alpha_z$ About z Axis: Application of moment $M_z$ causes the flexure to deflect through angle $\alpha_z$. Compliance is

$$\frac{\alpha_z}{M_z} = \frac{9\pi R^{1/2}}{2Ebt^{5/2}}$$

$$\sigma_{max} = \frac{6M_z}{bt^2}$$

$$\sigma_{max} = \frac{4}{3\pi} \frac{\alpha_z Et^{1/2}}{R^{1/2}}$$

How to design flexure hinges – Paros & Weisbord (1965)
FEA
Compare FEA with simple model.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>CALC</th>
<th>FEA</th>
<th>DIFFERENCE - CALCULATED RELATIVE TO FEA (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>beam width (mm)</td>
<td>3.00 3.00 3.00 3.00 3.00</td>
<td>3.00 3.00 3.00 3.00 3.00</td>
<td>alpha az</td>
</tr>
<tr>
<td>beam height (mm)</td>
<td>6.00 6.00 6.00 6.00 6.00</td>
<td>6.00 6.00 6.00 6.00 6.00</td>
<td>11% 6% 4% 1% -2%</td>
</tr>
<tr>
<td>hinge cut out diam (mm)</td>
<td>4.00 4.00 4.00 4.00 4.00</td>
<td>4.00 4.00 4.00 4.00 4.00</td>
<td>stress max</td>
</tr>
<tr>
<td>hinge thickness (mm)</td>
<td>0.10 0.20 0.30 0.40 0.50</td>
<td>0.10 0.10 0.15 0.20 0.25</td>
<td>13% 2% 1% -1% -2%</td>
</tr>
<tr>
<td>t/R</td>
<td>0.05 0.10 0.15 0.20 0.25</td>
<td>0.05 0.10 0.15 0.20 0.25</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Standard conditions</th>
<th>CALC 1</th>
<th>FEA</th>
<th>DIFFERENCE - CALCULATED RELATIVE TO FEA (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material Al alloy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Force applied at</td>
<td>0.05m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E (Youngs Mod)</td>
<td>7.10E+10Pa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F (Force)</td>
<td>0.1N</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M (Moment)</td>
<td>0.005Nm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tensile strength (approx)</td>
<td>280MPa</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
“FlexHinge”

http://en.vinksda.nl/toolkit-mechanical-calculations/calculating-flexure-hinges
Some design guidelines

• Displacement limited
  – Stress $\propto t^{1/2}$
  – Force $\propto t^{5/2}$

• Force limited
  - Stress $\propto 1 / t^2$
  - Displacement $\propto 1 / t^{5/2}$
Design considerations

• Stress ratio
  – < 15% of yield stress
• Monolithic or clamped
• Thermal effects
• Flexure kinematics
  – Idealised pin jointed mechanism
  – Distortion of pin jointed mech (point of hinge moves)

Ref. A.H Slocum. Precision Machine Design
I20 turbo slit flexure

- **Requirement**
  - Vertical slits scanned across horizontal fan (80 mm)
  - Light weight / small size due to requirements of fast scanning
  - Picomotor: 0.5” travel / 22N
  - To give 0.5mm total movement in \( dx \), by applying a force \( F \) in the \( y \) direction (Benefits resolution of slit position)

- **Geometry**
  - Rib length = 25.5mm
  - Web diam = 4mm, \( t = 0.3 \text{mm} \)
Design case study

1. Geometry
2. Apply simple flexure equations
3. Develop hinge
4. Develop overall geometry
5. Validate with FEA
Geometry

• For $\Delta x = 0.5$ mm,
  – $\Delta \theta = 11.31$ deg
  – $\Delta y = 5$ mm

• Moment at each hinge = $F / 4 \times 25$
Simple flexure equations

• From flexure equations
  – 16.4N to give $\Delta \theta = 11.3$ deg
• So for this movement:
  – Stress = 1921 MPa
    • Compare $\sigma_y$ for Titanium 4 = 484
  – Required load = 16.4N
  – Required travel = 5mm
    • Both are within the capabilities of the chosen PicoMotor
Revise geometry

• Force now applied directly in direction of required movement: +/- 0.25mm

• Apply flexure equation
  – Rib length $L = 40$, web thickness = 0.2 (hole = 5.0)
  – Force 0.411 N (OK for chosen motor – 22N) gives:
    • max stress 77 MPa ($\sigma_y = 484$: Design to ~ 15% of yield = 72.6)
    • deflection +/- 0.25mm (as required)
Validate with FEA
(for deflection 0.25mm, t/R = 0.08, angle = 6 mrad, material = Ti 4)

<table>
<thead>
<tr>
<th></th>
<th>Equation</th>
<th>Single beam FEA</th>
<th>Full FEA model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force (N)</td>
<td>0.411</td>
<td>0.477</td>
<td>0.472</td>
</tr>
<tr>
<td>Force (%)</td>
<td>-13%</td>
<td>1%</td>
<td>0%</td>
</tr>
<tr>
<td>Stress (MPa)</td>
<td>77.1</td>
<td>80.8</td>
<td>82.3</td>
</tr>
<tr>
<td>Stress (%)</td>
<td>-6%</td>
<td>-2%</td>
<td>0%</td>
</tr>
</tbody>
</table>
Implementation

Piezo actuator  Fixed slit  Moving slit  Flexure
Validation

Displacement (mm) vs. No of coins (at 9.5g each)

Series 1
Series 2
Series 3
Series 4
Series 5
In conclusion.

\[
\frac{\alpha_z}{M_z} = \frac{9\pi R^{1/2}}{2Ebt^{5/2}}
\]

\[
\sigma_{max} = \frac{6M_z}{bt^2}
\]

\[
\sigma_{max} = \frac{4\pi}{3} \frac{\alpha_zEt^{1/2}}{R^{1/2}}
\]

+ 

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